Mathematical Model on the dynamics of Lassa fever and the Application of High Order Stiffly Stable parameter dependent nested hybrid linear multistep methods

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Abstract- This paper is concerned with the formulation of a Mathematical model on the dynamics of Lassa fever and the subsequent application of a numerical method the “high order stiffly stable parameter dependent nested hybrid linear multistep methods” for the numerical simulation of the model. This model is stiff in nature. Various data were collected with respect to the particular species of rats carrying the virus that causes Lassa fever as well as infectious contacts made with those suffering from the disease. These are the significant aspect of the spread of the disease. The disease is highly infectious and has a high rate of spread. This model so developed is aimed at finding a way out of the spread of the disease. A numerical method is proposed for this model, MATHEMATICAL and MATLAB software’s were used to generate the parameters and the proposed method is A- stable and A(α)–stable for step number k ≤ 6.

Keywords: a special family, nested hybrid multi-step methods, A-stable, A (α)-stability, biological and epidemiological, dynamics of lassa fever.
AMS subject classification: 65L05, 65L06.

1.0 INTRODUCTION
Lassa fever is a viral haemorrhagic fever transmitted by rats. It has been known since the 1950s, but the virus was not identified until 1969, when two missionary nurses died from it in the town called Lassa in Nigeria. This disease is mostly common in West Africa [12] it can cause tens of thousands of death, even after recovery, the virus remains in body fluids, including semen [11]. The rat species Mastomys, in particular, M, natalensis serves as the vectors for the virus. This is a consistent host reservoir for the Lassa virus because of congenital neonatal infection, which results in rats with long-lasting and/or lifelong infection. Because of the mechanism of infection, there is no break in the natural chain from virus to host species [6]. The rats themselves might show no symptoms of the disease, but they shed the virus freely in urine and droppings, and secrete the virus in their saliva. Because certain varieties of rats often live in human homes, the virus is easily transmitted to humans. Transmission of the virus occurs via direct contact with rat urine, faeces and saliva; via contact with excretion or secretion-infected materials; or via ingestion of excretion-contaminated food. In some areas, the rodents are used as a food source, thus providing additional exposure to the infected rat blood, as well as allowing ingestion of potentially contaminated meat [8]. Unlike other viruses, Lassa virus can be fairly easily transmitted from human to human. Humans can contact the disease from other humans via aerosol transmission (coughing), or from direct contact with infected human blood, urine, or semen. The first symptoms of the disease typically occur 1-3 weeks after the patient comes into contact with the virus and can include increasingly high fever, sore throat, cough, eye inflammation (conjunctivitis), facial swelling, retrosternal pain (behind the breastbone), back pain, abdominal pains, vomiting, diarrhea and general weakness lasting for several days. Neurological symptoms have also been described, including hearing loss, tremors, and encephalitis (brain inflammation). The most common long-term complication of Lassa fever is deafness [13]. The mortality rates for Lassa fever are typically estimated at 15% to 20%. Some studies estimate mortality as high as 45%. One survey of Lassa infection v

Order Stiffly Stable
2.0 MODEL FORMULATION

The interest of this work on this disease grew from the fact that the researcher lives in Ekpoma, a town very close to IRRUA SPECIALIST TEACHING HOSPITAL. Lassa fever was first identified in this area when a medical doctor died, it was later discovered that the doctor died of this disease which he contacted in an attempt to treat a patient carrying the virus in this Hospital in 1982. After the death of this doctor, a Professor from Ambrose Alli University, Ekpoma, Professor Agbonlahor Omoike collected a sample of the common rats dominant in the area to the Laboratory where he discovered the virus. Since then, it has been a source of worry to the communities within the environment. Hence the interests to produce a mathematical model that can help identify the spread and subsequently suggest ways of curtailing the spread of the disease.

In the formulation of this model, the principle of Occam’s razor [1] were simply remembered, which is expressed in modern terms by Einstein, that “Everything should be made as simple as possible, but not simpler”. Though the model is simple, we strive to ensure that the basic variables involved in the disease dynamics are captured.

The model is \( \text{SET} \) where \( S \) stands for susceptible individuals, \( E \) stands for infected individuals and \( T \) stands for the population of the rodents.

The total population considered is \( N = S + E + T \). The proposed mathematical model is:

\[
\begin{align*}
\dot{S} &= \mu N - mS - \beta IS - \alpha ES + \gamma E, \quad S(0) = 0 \\
\dot{E} &= \beta IS + \alpha ES - \gamma E - mE, \quad E(0) = 0 \\
\dot{T} &= rT - \frac{rT^2}{K} - \phi T, \quad T(0) = 0
\end{align*}
\]

The symbols used in the model are as shown in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>Number of susceptible</td>
</tr>
<tr>
<td>( E )</td>
<td>Number of those who are infected with Lassa fever</td>
</tr>
<tr>
<td>( T )</td>
<td>Population of rodents carrying the virus</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Human birth rates (/ day)</td>
</tr>
<tr>
<td>( m )</td>
<td>Human death rates (/ day)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Pairwise infectious contacts rate with rodents (/ day)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Pairwise infectious rate with infected individuals (/ day)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Rate at which infected recover from lassa fever (/ day)</td>
</tr>
<tr>
<td>( R )</td>
<td>Growth rate of the rodents (/day)</td>
</tr>
<tr>
<td>( K )</td>
<td>Carrying capacity of the environment for the rodents</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Death rate of the rodents (/ day)</td>
</tr>
<tr>
<td>( N )</td>
<td>Total population of human</td>
</tr>
</tbody>
</table>

When modeling infectious diseases, there is the need to understand the dynamics of the disease stability analysis, this will help ascertain whether the disease will invade the community or not.

3.0 STABILITY ANALYSIS

To ascertain the behavior of the different populations in the community, there is the need to compute the linearization of the system of (1) above which can be obtained through the Jacobian matrix of the system. For the system of (1) above, the Jacobian matrix is:

\[
\begin{bmatrix}
-m - \beta I - \alpha E & \alpha S + \gamma & -\beta S \\
\beta I + \alpha E & \alpha S - \gamma - m & \beta S \\
0 & 0 & r - \frac{rT}{K} - \phi
\end{bmatrix}
\]

Equation (2) above is the computation of the stability analysis of the disease using Jacobian matrix. Applying the proposed numerical method the “high order stiffly stable parameter dependent nested hybrid linear multistep method” for the numerical simulation of the model i.e. “the dynamics of Lassa fever”, there is the need to show the derivation of the method for at least \( k = 1 \) in clear term and generalize for \( k = 2,3,4,5 \) and 6.

The general \( k \)-step of the proposed method is:
\[
Y_0 = \frac{\sum_{j=0}^{k} S_j y_{n+j} + h\omega_k f_{n+k}}{\sum_{j=0}^{k} E_j y_{n+j} + h\rho_j f(Y_i)}; \quad Y_0 = y_{n+1} \quad \text{(3)}
\]

\[
Y_{i+1} = y_{n+k-1} + \sum_{j=0}^{k-1} E_j y_{n+j} + h\rho_j f(Y_i); \quad i = 0(1) s-1; \quad Y_{i+1} = y_{n+1}\]

\[
y_{n+k} = y_{n+k-1} + \sum_{j=0}^{k-1} T_j y_{n+j} + h\theta_{s-1} f(Y_{s-1}) + h\lambda_k (f_{n+k} + a f_{n+k-1}); \quad 0 \leq c \leq k.
\]

In (3), the parameters \( S_j, T_j, \theta_{s-1}, E_j, \rho_j, \omega_k \) and \( \lambda_k \) with \( i = 1, \cdots, s-1, j = 0, 1, \cdots, k \) are all real coefficients. The \( f_{n+j} = f(x_{n+j}, y_{n+j}) \) is the first derivative function, \( h = x_{n+1} - x_n \) denotes the mesh size, \( k \) and \( s \) represents the step number, and the stage respectively, while \( Y_i = y(x_n + c_i h) + Oh^{s-1}, i = 0, \cdots, s-1, \) and \( y_{n+k} \) are the stage and the output point of the methods in (3). The \( f(Y_i) = \{f(x_n + c_i h, Y_i)\}^{s-1}_{i=0} \) denotes the derivative of the stages. The stability region of the formulas in (3) depend on the choice of the parameter \( a \). The \( c = [c_0, c_1, \cdots, c_s]^T \) is the abscissa vector of the input methods. The elements in \( c \) are computed from the abcissa generator: \( c_i = \left[ k - \frac{1}{2} \right]^{b-1}_{i=0}, k = 1, 2, \ldots, s-1. \)

**4.0 DERIVATION OF THE PROPOSED METHOD**

Using the general form of the \( k \)-step of the proposed method in (3) above, the method is derived for \( k, s \) by applying Taylor's series expansion to \( k, s \) respectively and by setting \( k = 1, s = 1, p = q + 1 = 3, c_0 = \frac{1}{2} \) and \( a = \frac{1}{2} \)

**The Method for \( k = 1 \)**

For \( k = 1 \), the input method \( Y_0 \) is derived using Taylor's series expansion;

\[
Y_0 = S_0 y_n + S_1 y_{n+1} + h\sigma_1 f_{n+1}
\]

where \( Y_0 = y(x_n + c_0 h) \)

\[
y_{n+1} = y(x_n + h)
\]

\[
f_{n+1} = y'(x_n + h)
\]

Substituting (5) into (4), yields;

\[
y(x_n + c_0 h) - S_0 y_n - S_1 y(x_n + h) - h\sigma_1 y'(x_n + h) = 0 \quad \text{(6)}
\]

Expanding (6) using Taylor's series expansion about \( x_n \), gives;

\[
-(-1 + S_0 + S_1) y_{x_n} + (c_0 - S_1 - \sigma_1) y'_{x_n} h + \frac{1}{2} (c_0^2 - S_1 - 2\sigma_1) y''_{x_n} h^2 + \frac{1}{6} (c_0^3 - S_1 - 3\sigma_1) y'''_{x_n} h^3 + \ldots = 0 \quad \text{(7)}
\]

Collecting the co-efficient of the power of \( h \), gives the following system of equations;

\[
-(-1 + S_0 + S_1) = 0
\]

\[
(c_0 - S_1 - \sigma_1) = 0
\]

\[
\frac{1}{2} (c_0^2 - S_1 - 2\sigma_1) = 0
\]

Solving the arising system of equations in (8) for the unknown values of \( S_0, S_1, \sigma_1 \), gives;

\[
S_0 = 1 - 2c_0 + c_0^2
\]

\[
S_1 = 2c_0 - c_0^2
\]

\[
\sigma_1 = c_0 + c_0^2
\]

Substituting the value of \( c_0 = 1/2 \) yields;

\[
S_0 = 1/4, S_1 = 3/4 \text{ and } \sigma_1 = -1/4
\]

Substituting the values into the input method \( Y_0 \) in (9) above gives;

\[
Y_0 = \frac{y_n}{4} + \frac{3}{4} y_{n+1} - \frac{h}{4} f_{n+1}
\]

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Expanding the output method \( y_{n+1} \) for \( k=1 \) using Taylor’s series expansion;

\[
y_{n+1} = I_0y_n + h\theta_0f(Y_0) + h\lambda_1(f_{n+1} + af_n)
\]  
(11)

Where

\[
y_{n+1} = y(x_n + h)
\]

\[
f(Y_0) = y'(x_n + c_0h)
\]

\[
f_{n+1} = y'(x_n + h)
\]

\[
f_n = y'_{x_n}
\]

Substituting (12) into (11), yields;

\[
y(x_n + h) - I_0y_n - h\theta_0y'(x_n + c_0h) - h\lambda_1[y'(x_n + h) + ay'_{x_n}] = 0
\]  
(13)

Expanding (13) using Taylor’s series expansion about \( x_n \), gives;

\[
(-1 + I_0)y_{x_n} - (-1 + \theta_0 + (1 + a)\lambda_1)y'_{x_n}h - \frac{1}{2}(-1 + 2c_0\theta_0 + 2\lambda_1)y''_{x_n}h^2 - \frac{1}{6}(-1 + 3c_0^2I_0 + 3\lambda_1)y'''_{x_n}h^3 + \ldots = 0
\]  
(14)

Collecting the co-efficient of the power of \( h \), gives the following system of equations;

\[
-(-1 + I_0) = 0
\]

\[
-(-1 + \theta_0 + (1 + a)\lambda_1) = 0
\]

\[
\frac{1}{2}(-1 + 2c_0\theta_0 + 2\lambda_1) = 0
\]  
(15)

Solving the arising system of equations in (15) for the unknown values gives;

\[
I_0 = 1, \quad \theta_0 = \frac{1-a}{2(-1+c_0+ac_0)} \quad \text{and} \quad \lambda_1 = \frac{1-2c_0}{2(-1+c_0+ac_0)}
\]

Substituting the value of \( c_0 = 1/2 \) and \( a = 1/2 \), yields;

\[
I_0 = 1, \quad \theta_0 = 1 \quad \text{and} \quad \lambda_1 = 0
\]

Inserting the values above into the output method in (11), gives;

\[
y_{n+1} = y_n + hf(Y_0)
\]  
(16)

Bringing the input method (10) and the output method (16) together, yields \( k = 1 \):

\[
y_0 = \frac{y_n}{4} + \frac{3}{4}y_{n+1} + \frac{h}{4}f_{n+1}
\]

\[
y_{n+1} = y_n + hf(Y_0)
\]  
(17)

In the same manner, we can also derive \( k = 2, 3, 4, 5 \) and \( 6 \).

### 5.0 NUMERICAL SIMULATIONS

The mathematical model in (1) above is now simulated using the high order stiffly stable parameter dependent nested hybrid linear multistep methods by applying the scheme in (17) for \( k=1 \) which is of order 3 and also independently derived through the use of Taylor’s series expansion.

\[
Y_0 = \frac{1}{4}(y_n + 3y_{n+1}) - \frac{h}{4}f_{n+1}; \quad q_0 = 2, \quad C_3 = \frac{1}{48}
\]  
(18)

\[
y_{n+1} = y_n + hf(Y_0); \quad p = 2, \quad C_3 = \frac{1}{24}
\]

Equation (18) produced values for the parameters estimated in the table below.

<table>
<thead>
<tr>
<th>Table 2: Parameters estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
</tr>
<tr>
<td>( r )</td>
</tr>
<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \alpha )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
<tr>
<td>( K )</td>
</tr>
</tbody>
</table>
Otunta et al (2007), Newton Raphson iterative method is used to resolve the implicitness in the methods.

\[
y_{n+k}^{[s+1]} = y_{n+k}^{[s]} - (F'(y_{n+k}^{[s]}))^{-1} F(y_{n+k}^{[s]}), \quad s = 0, 1, \ldots
\]  

(19)

where \( F'(y_{n+k}^{[s]}) \) is the Jacobian matrix. While an explicit one-step formula will be used to generate the starting value \( y_{n+1}^{[0]} \) for the iterative schemes

\[
y_{n+1}^{[0]} = y_{n} + \frac{h}{2} (f_{n-1} + f_{n})
\]

(20)

Fig. 1: Population size of susceptible individuals

Fig. 2: Population and rate of infected individuals

Fig. 3: Population size of Rodents
6. DISCUSSION AND CONCLUSION

The interest is to examine what happens in the community where the virus is prevalent and suggest ways of achieving a disease free community. Figure 1 to figure 3 gives the population size of individuals in the community as well as the rat population. From figure 1, it was observed that the community gets to a disease Free State as the population of the rodents reduces and this ultimately leads to a reduction in the population of infected individuals in figure 2. In this work, a Mathematical model for the dynamics of Lassa fever was presented. It was observed that to achieve a disease free environment and prevent an epidemic, there is the need to control the vectors causing the disease. This will mean eradicating the rodents population by a parametric projection aimed at eradicating the rats in the environment which agrees with the findings done by the Merlin Institute, London in [8]. However, most people will kick against this idea as the rats serve as meal for some people. Serious control of the rodent’s population is clearly advised in order to make the disease Free State achievable. Also, people should imbibe the culture of covering food always to prevent the rodents from defecating on it. In addition, a serious effort to reduce contacts with infected individuals is also suggested. This can be achieved by isolating the infected individuals as indicated in (1) until they get well from the disease. All this is necessary since human infection is due to contact with rodents or infected individuals and this will be made possible if human contact with the things infected by the carrier rodent is reduced to the barest minimum. However, to achieve a widespread prevention, there should be provision of vaccine to prevent the spread of Lassa fever to communities and the provision of basic amenities such as hospitals to our various communities for immediate treatment as the stability analysis carried out indicate that the disease is prevalent in the area of research as shown in the various diagrams.

REFERENCES: